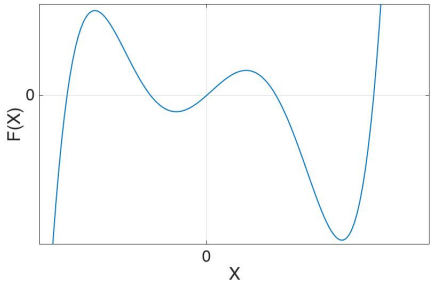
Homework 1: 1D analysis

1. Consider the equation: $\dfrac{dx}{dt} = a \cdot x - 5$.
   (a) Find the general solution of the system above.
   (b) What are the equilibrium points of the system?
2. Sketch the phase line, find all equilibrium solutions and determine their stability for:
   (a) $x' = x^2 + 8x$
   (b) $x' = x^2 - x^3$
   (c) $x' = \sin x$
   (d) $x' = \cos^2 x$
   (e) $x' = |1 - x^2|$
3. Consider the function graph:



   (a) Sketch the phase line corresponding to the function.
   (b) Determine stability of the steady states.
4. Consider the equation:
$$x' = x(1 - x) - h$$
   (a) Draw the phase line of the system corresponding to $h = 0$, $h = 1/4$, $h = 1/2$
   (b) Find the general solution of the equation (**optional** exercise, but gives an extra point).