Homework 3: Nonlinear systems

You may want to use `ode_vfield_2d.m` function drawing vector fields and sample trajectories for 2D ODE systems. Find enclosed.

1. Draw the major nullclines for the following systems (note: you need to draw *lines* at which RHS becomes effectively zero, you might want to double-check your drawings using Matlab's `contour` command plotting a zero *level* contour plot of function `z=RHS`, see `help contour`):

   (a) $x' = x^2 + y - 2$, $y' = y - x^2 + 1$.

   (b) $x' = x^2 + y^2 - 1$, $y' = -y - x$.

   (c) $x' = x(y - x + 2)$, $y' = y(3 - y + x)$.

   (d) $x' = y(-x^3 - y)$, $y' = x(2x - y)$.

   (e) $x' = y(x^2 - 2y)$, $y' = x(-y + x^3 - 2)$.

2. For each of the systems above, sketch the phase portrait by determining the orientation of the vector field.

3. Confirm that your sketch is correct by plotting the vector field in MATLAB using function `quiver` (you can use other numerical environment and the similar function therein). *Take only one system.*

   The function takes four arguments (read `help quiver`): `x`, `y` determine the phase space and `u`, `v` determine the velocity vectors. The usual way to produce grid of x-y values is:

   ```
   [x,y] = meshgrid(span);
   ```

   Choose `span` appropriately in order to see all equilibria or vary it to see different subparts of the phase space for better vector field resolution. For example, `span = -2:.2:2;` would make a grid with step 0.2 covering the square from -2 to 2 in both x and y directions.

   How to define the velocity vectors `u` and `v`?

4. Find the equilibria of the systems above and determine their stability using linear stability analysis. (*Hint:* you may want to use Matlab's `fsolve` or `fzero` to find the solutions of highly nonlinear functions, e.g. cubic.)

5. Consider the Brusselator ("chemical" oscillator) equations:

$$\begin{cases} x' = A + x^2 y - (B+1)x \\ y' = Bx - x^2 y \end{cases}$$

(a) Find the steady states of the system in general form, i.e. expressed in parameters A and B.

(b) Create a Matlab function describing the Brusselator system for `ode45` solver. *Hint:* you can write the function in **a file** or use **anonymous** functions; importantly, the input of the function must be `(t,x)`, where `x` is a vector of variables (`t` is a phony variable not used in autonomous systems), whereas the output must be a column vector of RHS's of the system. Finally, you can see `help ode45` and related topics.

(c) The Hopf bifurcation criterion is $B = 1 + A^2$, that is, if $B < 1 + A^2$ there is a stable focus, but if $B > 1+A^2$, the limit cycle emerges. Put $A = 1$, $B = 1$ and simulate the system using `ode45` Matlab solver (put any positive initial conditions). Do the steady state values of x and y coincide with ones you have calculated above?

(d) Now put $A = 1$ and $B = 3$. Can you see oscillations of x and y?

(e) Plot the phase portrait, that is x–y plane. Can you see the limit cycle?