Homework 5: Hamiltonian vs. Dissipative

Find in the enclosed material file `let.zip` for the Lyapunov exponents toolbox.

1. Planar Hamiltonian systems have equilibria characterized by eigenvalues with opposite signs. In other words, there are only saddles and center points possible. Prove this statement for an autonomous Hamiltonian $H(x, y)$, which is differentiable on both variables as many times as you wish.

2. Consider the ideal pendulum:

$$\begin{cases} \theta' = \nu \\ \nu' = -sin\theta \end{cases}$$

   (a) Write a Hamiltonian function $H(\theta, \nu)$ for this system.

   (b) Plot its surface and level lines $H(\theta, \nu) = Constant$ (use `mesh` and `contour` commands, respectively). *Hint*: define the (θ, ν) space using `meshgrid`; use at least 40 levels with `contour`, see `help contour`.

   (c) The system moves on the lines $H(\theta, \nu) = Constant$. Using this fact and general idea about the pendulum motion, explain the behaviour of the system.

3. Download and unzip `let.zip` file with LET toolbox into a folder where the Matlab session is currently in (`Current folder`) or anywhere under the Matlab path.

   (a) We will use the Rössler system:

$$\begin{cases} x' = -y - z \\ y' = x + ay \\ z' = b + z(x - c) \end{cases}$$

   (b) In the `LET` folder find the file `rossler_void.m`. It has empty spaces marked with `??`. Fill in the spaces with correct expressions to model the Rössler system: parameters ($a = 0.15$, $b = 0.20$, and $c = 10.0$), RHS's of the system, and Jacobian `J`.

   (c) Type in `let` in the Matlab command line.

(d) Run the LET main program and select `Setting`.

(e) Under the `Setting` write the `ODE function` name, i.e. `rossler_void`; `Initial Condition(s)` — `1 1 1`; `No. of linearized ODEs` — `9`; `Absolute tolerance` — `1e-06`. Press `OK`.

(f) Start the simulation and observe calculation of the Lyapunov spectrum. The values of λ_i will be printed at the window's bottom as well as the Lyapunov dimension D_L. You may want to stop simulation if you feel the calculation converged.

(g) Report λ_i and D_L.

(h) Create a Matlab function describing the Rössler system for `ode45` solver.

(i) Simulate the system with the aforementioned parameters using `ode45` and plot the Rössler attractor.