Homework 9: Excitability phenomena. Neurons.

Enclosed material: ***stoch_if.m**, **hh1.m**, **hh2.m**, **hh3.m**, **hh4.m**.*

1. Integrate-and-fire model of a neuron looks like:

   $$C\frac{dV}{dt} = I - g_l(V - E_l)$$

   where C is the membrane capacity, I is the applied current, g_l and E_l are conductance and Nernst equilibrium potential for the leakage currents. Given $E_l = 0$, $R = 1/g_l$, and the membrane time constant $\tau_m = RC$ we can rewrite this equation as:

   $$\tau_m\frac{dV}{dt} = -V + RI$$

   To see the neuronal firing and the "threshold" phenomenon one needs to integrate this equation, resetting the membrane potential $V(t)$ to its initial value, whenever it reaches the threshold (exactly what `stoch_if.m` file does).

   (a) Run the model to see the firing.

   (b) Decrease the amplitude of the input current `A_I_ext` from 8.0 to 6.0. Do you still see spikes?

   (c) Decrease the threshold `u_threshold` to see the spikes again (say to 10).

   (d) You can apply input of different shapes. Uncomment the code for `I_ext` defined with sine function. Run the simulation.

   (e) You can also apply some noise to the model (set the noise amplitude `sigma` equal to 0.1). You can further increase `sigma` to observe how regularity of the spikes breaks with increased amplitude of noise.

2. Explain what it means when a current is referred to as *persistent*, *transient*, and *Ohmic*

3. Look into the file `hh1.m`. This script simulates the Hodgkin-Huxley model of a neuron. Notice how nonlinear are the `alpha` and `beta` functions, defining the dynamics of the gating particles n, m, and h (array `x`).

   (a) Simulate the model. During the simulation the external current of $10\,\mu\text{A/cm}^2$ amplitude is applied (look for the place in the file, where it is done).
   (b) Apply other amplitudes of the input current: 5, 3, 1, -1, -5, -10 $\mu\text{A/cm}^2$. Report values of the input, for which you *cannot* see the spike (*Warning*: look both at the shape of the spike and its amplitude, compare the amplitudes for different cases).

4. Next we apply two current injections (one weak, one strong) to observe the "below-" and "above-threshold" responses. We also plot all currents of the model along with other variables to see how they change during the spike development (this is implemented in `hh2.m`).
   (a) Find the place where currents `I` for Na^+, K^+, and leak are defined (marked with `???`). Define `I` according to the Ohm law, given conductances `gnmh`, Nernst potentials `E` and membrane potential `V`. (*Remember*: `gnmh` and `E` are 1x3 vectors, defining corresponding entities for Na^+, K^+, and leak).
   (b) Simulate the model to see the output.

5. Finally, we can apply ramping current injections, both increasing and decreasing with a constant slope (see files `hh3.m` and `hh4.m`).
   (a) Simulate the model with the increasing/decreasing input current ramp.
   (b) Initially the increasing current causes slow depolarization of the membrane. After some critical value the tonic spiking activity emerges $\Rightarrow$ bifurcation. What is the bifurcation type?
   (c) With the decreasing current application the picture is reversed. Does it help to understand the bifurcation type? (*Hint*: try to estimate from the two graphs the input current value, at which spiking emerges/disappears).