Homework 10: Coupled systems.

1. Implement the system of two coupled Brusselators for direct simulation with e.g. `ode45`. Recall the system is:

$$\begin{cases} I \begin{cases} \dfrac{dx_1}{dt} = A - (B+1)x_1 + x_1^2 y_1 \\[2ex] \dfrac{dy_1}{dt} = Bx_1 - x_1^2 y_1 + D(y_2 - y_1) \end{cases} \\[4ex] II \begin{cases} \dfrac{dx_2}{dt} = A - (B+1)x_2 + x_2^2 y_2 \\[2ex] \dfrac{dy_2}{dt} = Bx_2 - x_2^2 y_2 + D(y_1 - y_2) \end{cases} \end{cases}$$

   (a) Matlab's `ode45`-like numerical integrators accept the initial condition vector `x0`. What is the dimension of this vector?

   (b) We need the random initial conditions technique to locate certain attractors amongst other coexisting attractors. Create uniformly distributed random initial conditions in the specified ranges of variables with $x_0 = x_{min} + (x_{max} - x_{min}) \cdot U(0,1)$, where $U(0,1)$ is a uniformly distributed random number between 0 and 1 (use `rand`). Create 20–30 initial condition sets in the range $[0,5]$ for all variables.

   (c) Set parameters as $A = 1$, $B = 2.5$, $D = 0.5$. Then simulate the system for each vector of the random initial conditions (take final time 100 or more). Plot the result of each simulation in the same window. Make sure you plot only **one** variable! Report the figure. How many distinct dynamical regimes do you see?

   (d) Select the initial conditions that lead to the steady state dynamics (*Hint*: compare the difference between the last two points of the trajectory to some small number: for a steady state the difference will be small, given enough time of integration). Plot the trajectory for one of these initial conditions for all four variables. Report the figure. Name the dynamical regime.

   (e) Now simulate the system for 500 initial conditions (do not plot trajectories now, only detect the steady state regimes using the procedure developed at the previous point above). How many out of the 500 lead to the steady state dynamics?